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APPROXIMATE FORMULAS FOR VISCOSITY AND THERMAL CONDUCTIVITY OF GAS MIXTURES

by Richard S. Brokaw

Lewis Research Center
Cleveland, Ohio

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CONDUCTIVITY OF GAS MIXTURES

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SUMMARY

Approximate expressions for the viscosity and thermal conductivity of monatomic gas mixtures have been derived from the rigorous kinetic theory formulas for binary mixtures. The approximations, which are of the Sutherland-Wassiljewa form, are compared with rigorous calculations for binary and ternary mixtures of the noble gases. These approximations are very accurate: maximum errors are three parts in ten thousand for viscosity (He-Ne-Ar at 20° C) and seven parts in ten thousand for thermal conductivity (He-Kr-Xe at 29° C); the corresponding root-mean-square errors for all the calculations are 0.015 percent for viscosity and 0.036 percent for thermal conductivity. Thus, these approximations seem adequate for all practical applications; indeed, if greater accuracy is required, the higher Chapman-Enskog approximations should be considered as well.

INTRODUCTION

Approximate formulas for the viscosity and thermal conductivity of gas mixtures were derived by Sutherland and Wassiljewa six decades or more ago (refs. 1 and 2). These derivations, which are based on simple mean-free-path arguments, lead to expressions of the form

$$\mathscr{P}_{mix} = \sum_{i=1}^{\nu} \{\mathscr{P}_i\} = \sum_{i=1}^{\nu} \frac{x_i \mathscr{P}_i}{x_i + \sum\limits_{\substack{j=1 \\ j \neq i}}^{\nu} A_{ij} x_j} \qquad (1)$$

Here $\mathscr{P}_{mix}$ is the viscosity or conductivity of the gas mixture, while the quantity $\{\mathscr{P}_i\}$ may be thought of as a partial viscosity or conductivity due to component i (analogous to a partial pressure), and $\mathscr{P}_i$ is the property of the pure component. The x_i are the mole fractions that specify the gas composition and the A_{ij} are parameters presumed independent of composition.

Equation (1) has intrigued a number of investigators over the years, because of its simple analytic form and because it represents experimental data extremely well, provided the A_{ij} are suitably chosen. (Often an extensive range of pairs of A_{ij} and A_{ji} give satisfactory agreement for binary mixtures.) Numerous attempts have been made to develop, either empirically or theoretically, generalized expressions for the A_{ij}. Some of the more successful empirical efforts include the formulas of references 3 to 5 for viscosity and references 6 and 7 for thermal conductivity. These results were obtained with at least a qualitative regard for simple kinetic theory and dimensional considerations.

The rigorous Chapman-Enskog theory formulas for the viscosity (ref. 8, p. 531) and thermal conductivity (ref. 9) of mixtures of monatomic gases are of the form

$$\mathscr{P}_{mix} = - \frac{\begin{vmatrix} a_{11} & \cdots & a_{1\nu} & x_1 \\ \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdots & \cdot & \cdot \\ a_{1\nu} & \cdots & a_{\nu\nu} & x_\nu \\ x_1 & \cdots & x_\nu & 0 \end{vmatrix}}{\begin{vmatrix} a_{11} & \cdots & a_{1\nu} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{1\nu} & \cdots & a_{\nu\nu} \end{vmatrix}} \tag{2}$$

The quantities a_{ij} are functions of composition (and are different for viscosity and thermal conductivity).

The link between equations (1) and (2) was first provided by Hirschfelder, Curtiss, and Bird (ref. 8, p. 532) who pointed out that equation (2) may be expanded as a series. By discarding all but the first terms, they obtained an expression for mixture viscosity of the form of equation (1); in fact, their A_{ij} are proportional to the A_{ij} of Buddenberg and Wilke (ref. 3), with a numerical factor of two in place of the empirical factor of 1.385 of Buddenberg and Wilke. Mason and Saxena (ref. 10) carried out the analogous expansion for thermal conductivity. After some empirical adjustments to compensate for the neglect of higher terms in the expansion of equation (2), they proposed an expression for the A_{ij} which is simply 1.065 times the A_{ij} for viscosity developed by Wilke (ref. 4). (In more recent papers Mason, et al. (refs. 11 and 12) omit the factor 1.065.)

In reference 13, the present author developed approximations to equation (2) that take some account of the higher terms in the series expansion (or, equivalently, take some account of the off-diagonal a_{ij} of equation (2)). These approximations may be written

$$\mathscr{P}_{mix} = \sum_{i=1}^{\nu} \{\mathscr{P}_i\}_I \, C_i \tag{3}$$

where the $\{\mathscr{P}_i\}_I \equiv x_i^2\left(\sum_{j=1}^{\nu} a_{ij}\right)^{-1}$ prove to be of the same form as the partial properties in equation (1), and the C_i are correction terms. If the C_i are taken as unity, a first approximation

$$\mathscr{P}_{I,mix} = \sum_{i=1}^{\nu} \{\mathscr{P}_i\}_I \tag{4}$$

is obtained which is in the form of equation (1). The C_i can be chosen so as to make equation (3) rigorous for binary mixtures and an excellent approximation indeed for multicomponent mixtures (within a few parts per thousand). These higher approximations, however, are only of academic interest since all the elements required for the rigorous expression (eq. (2)) must be computed; further the ratio of symmetric determinants in equation (2) is relatively easy to reduce in specific numerical cases.

Viscosities and conductivities calculated from equation (4) are consistently larger than rigorously computed values, by as much as 8 percent. Indeed, recently it has been shown that equation (4) provides an upper bound to the rigorous theoretical value (eq. (2)) (ref. 14 and a private communication from J. M. Yos of Avco Research and Advanced Development Division). This systematic error can often be counteracted by introducing a further approximation. The expressions for the A_{ij} contain a quantity λ_{ij} (defined in ref. 8, p. 531, by eq. (8.2-34)) that characterizes the interaction between unlike molecules. If one assumes a rigid-sphere model, the λ_{ij} can be eliminated, and the A_{ij} expressed in terms of the properties of the pure components only (ref. 15). In general, this introduces a systematic error in the opposite direction and hence reduces the overall error, except in cases where the mass ratio is very large (e.g., He-Xe mixtures). Since in this approximation the A_{ij} involve only the pure component properties they can conveniently be obtained from alignment charts (ref. 16).

The physical significance of equations (1) and (2) has been discussed in an interesting series of papers by Cowling, Gray, and Wright (refs. 14, 17, and 18). They recognize two principal phenomena in the transport of heat or momentum through gas mixtures. The major effect is that molecules of one species impede the transport of heat or momentum by the other species in the mixture. This effect is accounted for by the diagonal elements of the determinants in equation (2) (the a_{ii}). The second effect is an enhancement, due to the transfer of heat or momentum from one species to another, accounted for by the off-diagonal a_{ij}. Thus, formulas of the form of equation (1) are most simply obtained (ref. 8, p. 532) by neglecting the transfer of transport.

In view of the large body of literature on approximate expressions for mixture viscosity and conductivity, it is perhaps in order to inquire as to what possible benefits can accrue from any new formulations. The most precise experimental data on mixture viscosity are in close accord with rigorous theory (ref. 19), whereas several of the approximate schemes show errors amounting to several percent in some cases. One might hope, then, to develop improved approximations that more faithfully represent both theory and experiment. The situation with regard to the heat conductivity is less clear. Experimental data are generally of rather low precision and accuracy, with apparent errors amounting to several percent in many instances. Thus, it would require a rather careful statistical comparison with a large number and variety of experimental data to make a meaningful selection among the approximations already at hand. It is easier, and perhaps more significant, to test these schemes as approximations to the rigorous theory. In this regard there is again room for improvement.

Finally, any new approximate formulas should be no more complex than equation (1). Equation (1) appears deceptively simple, but can be tedious to use if there are many gases in the mixture. Thus, for a mixture of ν components, one must compute $\nu(\nu - 1)$ values of A_{ij}, whereas the rigorous formulas require only half again as many terms characterizing interactions between unlike molecules. Thus, expressions more complex than equation (1) are unlikely to offer much computational advantage over the rigorous formulas.

This report presents improved approximations to the rigorous Chapman-Enskog formulas for the viscosity and heat conductivity of gas mixtures. These expressions are derived from the rigorous binary-mixture formulas by a well-defined approximation and with no empirical adjustments. The new expressions are compared with rigorous calculations for binary and ternary mixtures of monatomic gases. They are superior to all previous approximations; indeed, the errors are of the order of the differences between the first and higher Chapman-Enskog approximations. Hence, the present approximations are suitable for all applications, unless extremely high accuracy is required.

DERIVATION OF APPROXIMATE FORMULAS

The rigorous formulas for the viscosity and thermal conductivity of binary mixtures of monatomic gases can be expressed (ref. 13) by equation (3), with

$$C_i^{-1} = 1 + \frac{b_{ij}\left(\{\mathscr{P}_i\}_I \frac{1}{x_i} - \{\mathscr{P}_j\}_I \frac{1}{x_j}\right)x_j}{1 + b_{ij}\{\mathscr{P}_j\}_I \frac{1}{x_j}}$$

$$\equiv 1 + B_{ij}x_j \tag{5}$$

where $b_{ij} \equiv - a_{ij}(x_i x_j)^{-1}$ is a function independent of composition. Thus, the terms of equation (3) may be written

$$\{\mathscr{P}_i\} = \{\mathscr{P}_i\}_I C_i = \frac{\mathscr{P}_i x_i}{(x_i + A'_{ij}x_j)(1 + B_{ij}x_j)}$$

$$= \frac{\mathscr{P}_i x_i}{x_i + \left[A'_{ij} + (x_i + A'_{ij}x_j)B_{ij}\right]x_j} \tag{6}$$

Here A'_{ij} is the expression derived in reference 13 (designated as φ_{ij} for viscosity and ψ_{ij} for thermal conductivity). Equation (6) is of the form of equation (1), except that the A_{ij} are now a function of composition:

$$A_{ij} = A'_{ij} + (x_i + A'_{ij}x_j)B_{ij}$$

$$= A'_{ij} + \frac{b_{ij}\left(\mathscr{P}_i - \mathscr{P}_j \frac{x_i + A'_{ij}x_j}{A'_{ji}x_i + x_j}\right)}{1 + b_{ij}\left(\frac{\mathscr{P}_j}{A'_{ji}x_i + x_j}\right)} \tag{7}$$

The first term of equation (7) is the larger, and the second does not change drastically with composition. Consequently, as Wright and Gray (ref. 18) have observed, equation (7) is quite insensitive to composition. Thus, if equation (7) is evaluated at some intermediate composition, an excellent approximation over the entire range may be expected. A convenient choice is to let

$$x_i = \frac{\sqrt{A'_{ij}}}{\sqrt{A'_{ij}} + \sqrt{A'_{ji}}} \tag{8}$$

This is a composition weighted toward the lighter component, where both the viscosity and heat conductivity of mixtures of light and heavy gases undergo the most abrupt variation with composition. Thus, from equations (7) and (8)

$$A_{ij} \cong A'_{ij} + \frac{b_{ij}\left(\frac{\mathscr{P}_i}{\sqrt{A'_{ij}}} - \frac{\mathscr{P}_j}{\sqrt{A'_{ji}}}\right)\sqrt{A'_{ij}}}{1 + b_{ij}\left(\frac{\sqrt{A'_{ij}} + \sqrt{A'_{ji}}}{1 + \sqrt{A'_{ij}A'_{ji}}}\right)\frac{\mathscr{P}_j}{\sqrt{A'_{ji}}}} \tag{9}$$

Equation (9) has been derived for binary mixtures but should apply reasonably well to multicomponent mixtures too. In the following sections the explicit expressions for the A_{ij} for viscosity and thermal conductivity are presented and tested by computing properties of binary and ternary mixtures of the noble gases.

VISCOSITY OF GAS MIXTURES

For mixture viscosity, equation (9) becomes

$$\varphi_{ij_{II}} = \varphi_{ij} + \frac{\left(M_i\sqrt{\varphi_{ij}} - M_j\sqrt{\varphi_{ji}}\right)\sqrt{\varphi_{ij}}}{\frac{3A^*_{ij}(M_i + M_j)}{5 - 3A^*_{ij}} + \frac{\sqrt{\varphi_{ij}} + \sqrt{\varphi_{ji}}}{1 + \sqrt{\varphi_{ij}\varphi_{ji}}} M_j\sqrt{\varphi_{ji}}} \tag{10}$$

where M_i and M_j are the molecular weight of components i and j, and A^*_{ij} (defined in ref. 8, p 531, eq. (8.2-15)) is a number close to unity whose exact value depends on the nature of the intermolecular potential and the temperature. Furthermore,

$$\varphi_{ij} \equiv \frac{\lambda_i}{\lambda_{ij}} = \frac{\eta_i}{\eta_{ij}}\left(\frac{2M_j}{M_i + M_j}\right) \tag{11}$$

with λ_i the monatomic thermal conductivity of component i ($= (15/4)(R/M_i)\eta_i$ for a polyatomic gas) and η_i is the viscosity of component i. The quantities λ_{ij} and η_{ij} characterize the interaction between unlike molecules and are defined in reference 8 (p. 534 eq. (8.2-34) and p. 529 eq. (8.2-20)). (Note that $\varphi_{ij} = A'_{ij}$ for viscosity.) Equation (10) can be simplified somewhat by noting that for realistic intermolecular potentials A^*_{ij} is often approximately 10/9, and that $\left(\sqrt{\varphi_{ij}} + \sqrt{\varphi_{ji}}\right)\left(1 + \sqrt{\varphi_{ij}\varphi_{ji}}\right)^{-1}$ is roughly unity, so that

$$\varphi_{ij_{II}} \cong \varphi_{ij} + \frac{M_i\sqrt{\varphi_{ij}} - M_j\sqrt{\varphi_{ji}}}{2(M_i + M_j) + M_j\sqrt{\varphi_{ji}}}\sqrt{\varphi_{ij}} \tag{12}$$

Approximate viscosities for the helium-neon-argon system, computed from equations (1) and (10) or (12) are compared with values calculated by the rigorous Chapman-Enskog theory in tables I and II. All calculations are based on the exponential-6 potential and the force constants of reference 20. Also shown are the experimental measurements of references 21 and 22.

From tables I and II it is clear that equations (1) and (10) are extremely accurate indeed, with a maximum error of only three parts in ten thousand. The errors seem to be systematically positive, although more calculations or analysis would be needed to prove this generally true. Thus, it appears that equations (1) and (10) are adequate for all practical applications; indeed, if greater accuracy is required the higher Chapman-Enskog approximations should be considered as well.

When equation (12) is used in place of equation (10) errors are increased by about an order of magnitude, but the accuracy remains acceptable for most purposes. Thus, equation (12) is entirely adequate in those frequent cases where there is no detailed information about the pairwise interactions between unlike molecules; in other words, in those cases where empirical combining rules are used to estimate the potential between unlike molecules.

A statistical comparison of the present approximations with previous methods is shown in table III. The present approximations are clearly superior, although the approximation of reference 5 is better than the others, roughly by an order of magnitude. No clear choice between the approximations of references 3, 4, and 13 can be made on the basis of these calculations. (Note that the errors for the method of ref. 13 are always positive, in accord with theory, ref. 14.)

THERMAL CONDUCTIVITY OF MONATOMIC GAS MIXTURES

For mixture thermal conductivity, equation (9) becomes

$$\psi_{ij_{II}} = \psi_{ij} + \frac{\left(\frac{\sqrt{\psi_{ij}}}{F_{ij}} - \frac{\sqrt{\psi_{ji}}}{F_{ji}}\right)\sqrt{\psi_{ij}}}{\frac{(M_i + M_j)^2}{M_i M_j\left(\frac{55}{8A^*_{ij}} - \frac{3B^*_{ij}}{2A^*_{ij}} - 2\right)} + \left(\frac{\sqrt{\psi_{ij}} + \sqrt{\psi_{ji}}}{1 + \sqrt{\psi_{ij}\psi_{ji}}}\right)\frac{\sqrt{\psi_{ji}}}{F_{ji}}} \tag{13}$$

where

$$\psi_{ij} \equiv \varphi_{ij} F_{ij}$$

and

$$F_{ij} \equiv 1 + \frac{M_i - M_j}{(M_i + M_j)^2}\left[\left(\frac{15}{4A^*_{ij}} - 1\right)(M_i - M_j) + \left(\frac{3B^*_{ij}}{2A^*_{ij}} + \frac{5}{8A^*_{ij}}\right)M_j\right]$$

Here B^*_{ij} (defined in ref. 8, p. 531 eq. (8.2-16)), like A^*_{ij}, is a number close to unity, whose exact value depends again on the intermolecular potential and the temperature. (Note that $\psi_{ij} = A^{*}_{ij}$ for thermal conductivity.) Equation (13) can be simplified by letting $\left(\frac{55}{8A^*_{ij}} - \frac{3B^*_{ij}}{2A^*_{ij}} - 2\right) \approx 2.6$ and $\left(\sqrt{\psi_{ij}} + \sqrt{\psi_{ji}}\right)\left(1 + \sqrt{\psi_{ij}\psi_{ji}}\right)^{-1} \approx 1$, with the result

$$\psi_{ij_{II}} \cong \psi_{ij} + \frac{\frac{\sqrt{\psi_{ij}}}{F_{ij}} - \frac{\sqrt{\psi_{ji}}}{F_{ji}}}{\frac{(M_i + M_j)^2}{2.6\ M_iM_j} + \frac{\sqrt{\psi_{ji}}}{F_{ji}}} \sqrt{\psi_{ij}} \tag{14}$$

Approximate conductivities for the helium-krypton-xenon system are compared with rigorous calculations in tables IV and V. Calculations are again based on the exponential-6 potential, with force constants for the helium-helium, helium-xenon, and xenon-xenon interactions from reference 20, and force constants for the helium-krypton, krypton-krypton, and krypton-xenon interactions from reference 23. The experimental measurements of reference 24 are shown as well.

Tables IV and V show that equations (1) and (13) approximate the rigorous calculations very closely. The maximum error of seven parts in ten thousand is somewhat larger than observed with the analogous viscosity approximation (eqs. (1) and (10)). This large error is perhaps a consequence of the wide conductivity variation - a factor of 26 compared with a factor 1.6 for viscosity. Once again, errors seem to be systematically positive. The conclusion is the same as for mixture viscosity: equations (1) and (13) are suitable for all practical purposes, and if greater accuracy is desired the higher Chapman-Enskog approximations must be considered.

When equation (14) is used in place of equation (13), errors are only slightly increased. This is perhaps fortuitous; it would be prudent to anticipate larger errors - probably a few tenths of a percent in some cases - by analogy with mixture viscosity (eqs. (1) and (12)). Again, equation (14) should suffice when information about the interactions between unlike molecules is lacking. In addition, equations (1) and (14) should be suitable for calculating the translational contribution to the thermal conductivity of polyatomic gas mixtures when Hirschfelder's Eucken-type approximation is used (refs. 25 and 26). In this event, neglecting the effects of inelastic collisions (ref. 27) will certainly lead to much larger errors than those caused by using equation (14) in place of equation (13).

Table VI presents a statistical comparison of previous approximations and the present methods. Equations (1) and (13) or (14) are vastly superior with errors smaller by two orders of magnitude. (Once again the errors for the method of ref. 13 are all positive.)

CONCLUDING REMARKS

These approximate expressions for mixture viscosity (eq. (10)) and thermal conductivity of monatomic gas mixtures (eq. (13)) are so accurate that they may well be regarded as the "correct" coefficients for the Sutherland-Wassiljewa equation (eq. (1)). As such they represent a logical point of departure for more convenient approximations; equations (12) and (14) represent a first step in this direction.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, July 31, 1964

REFERENCES

1. Sutherland, William: The Viscosity of Mixed Gases. Phil. Mag., vol. 40, 1895, pp. 421-431.

2. Wassiljewa, A.: Heat-Conduction in Gaseous Mixtures. Physik Zs., vol. 5, 1904, pp. 737-742.

3. Buddenberg, J. W., and Wilke, C. R.: Calculation of Gas Mixture Viscosities. Ind. and Eng. Chem., vol. 41, no. 7, July 1949, pp. 1345-1347.

4. Wilke, C. R.: A Viscosity Equation for Gas Mixtures. Jour. Chem. Phys., vol. 18, no. 4, Apr. 1950, pp. 517-519.

5. Francis, W. E.: Viscosity Equations for Gas Mixtures. Trans. Faraday Soc., vol. 54, 1958, pp. 1492-1497.

6. Lindsay, Alexander L., and Bromley, LeRoy A.: Thermal Conductivity of Gas Mixtures. Ind. and Eng. Chem., vol. 42, no. 8, Aug. 1950, pp. 1508-1511.

7. Cheung, Henry, Bromley, LeRoy A., and Wilke, C. R.: Thermal Conductivity of Gas Mixtures. A.I.Ch.E. Jour., vol. 8, no. 2, May 1962, pp. 221-228.

8. Hirschfelder, Joseph O., Curtiss, Charles F., and Bird, R. Byron: Molecular Theory of Gases and Liquids. John Wiley & Sons, Inc., 1954, p. 531.

9. Muckenfuss, Charles, and Curtiss, C. F.: Thermal Conductivity of Multicomponent Gas Mixtures. Jour. Chem. Phys., vol. 29, no. 6, Dec. 1958, pp. 1273-1277.

10. Mason, E. A., and Saxena, S. C.: An Approximate Formula for the Thermal Conductivity of Multicomponent Gas Mixtures. Phys. of Fluids, vol. 1, no. 5, Sept.-Oct. 1958, pp. 361-369.

11. Vanderslice, J. T., Weissman, Stanley, Mason, E. A., and Fallon, R. J.: High-Temperature Transport Properties of Dissociating Hydrogen. Phys. of Fluids, vol. 5, no. 2, Feb. 1962, pp. 155-164.

12. Yun, Kwang-Sik, Weissman, Stanley, and Mason, E. A.: High-Temperature Transport Properties of Dissociating Nitrogen and Dissociating Oxygen. Phys. of Fluids, vol. 5, no. 6, June 1962, pp. 672-678.

13. Brokaw, Richard S.: Approximate Formulas for the Viscosity and Thermal Conductivity of Gas Mixtures. Jour. Chem. Phys., vol. 29, no. 2, Aug. 1958, pp. 391-397.

14. Cowling, T. G., Gray, P., and Wright, P. G.: The Physical Significance of Formulae for the Thermal Conductivity and Viscosity of Gaseous Mixtures. Proc. Roy. Soc. (London), ser. A, vol. 276, no. 1364, Nov. 5, 1963, pp. 69-82.

15. Brokaw, R. S.: Energy Transport in High Temperature and Reacting Gases. Planetary and Space Sci., vol. 3, 1961, pp. 238-252.

16. Brokaw, Richard S.: Alignment Charts for Transport Properties, Viscosity, Thermal Conductivity and Diffusion Coefficients for Nonpolar Gases and Gas Mixtures at Low Density. NASA TR R-81, 1961.

17. Cowling, T. G.: The Theoretical Basis of Wassiljewa's Equation. Proc. Roy. Soc. (London), ser. A, vol. 263, no. 1313, Sept. 5, 1961, pp. 186-187.

18. Wright, P. G., and Gray, Peter: Collisional Interference Between Unlike Molecules Transporting Momentum or Energy in Gases. Trans. Faraday Soc., vol. 58, no. 469, Jan. 1962, pp. 1-16.

19. Brokaw, Richard S.: Transport Properties of Dilute Gas Mixtures. Proc. Int. Seminar on Transport Properties of Gases, Brown Univ., Jan. 20-24, 1964, pp. 97-121.

20. Mason, Edward A.: Forces Between Unlike Molecules and the Properties of Gaseous Mixtures. Jour. Chem. Phys., vol. 23, no. 1, Jan. 1955, pp. 49-56.

21. Trautz, Max, and Binkele, H. E.: Viscosity, Heat Conductivity and Diffusion in Gaseous Mixtures. VIII. The Viscosity of Hydrogen, Helium, Neon and Argon and Their Binary Mixtures. Ann. Physik, ser. 5, vol. 5, 1930, pp. 561-580.

22. Trautz, Max, and Kipphan, Karl F.: Viscosity, Heat Conductivity and Diffusion in Gaseous Mixtures. IV. The Viscosity of Binary and Ternary Mixtures of Noble Gases. Ann. Physik, ser. 5, vol. 2, 1929, pp. 743-748.

23. Mason, Edward A., and von Ubisch, Hans: Thermal Conductivities of Rare Gas Mixtures. Phys. of Fluids, vol. 3, no. 3, May-June 1960, pp. 355-361.

24. von Ubisch, Hans: The Thermal Conductivities of Mixtures of Rare Gases at 29° C and 520° C. Arkiv Fysik, vol. 16, no. 7, 1959, pp. 93-100 (numerical values cited in ref. 23).

25. Hirschfelder, Joseph O.: Heat Conductivity in Polyatomic, Electronically Excited, or Chemically Reacting Mixtures, III. Sixth Symposium (International) on Combustion, Reinhold Pub. Corp., 1957, pp. 351-366.

26. Hirschfelder, Joseph O.: Generalization of the Eucken Approximation for the Heat Conductivity of Polyatomic or Chemically Reacting Gas Mixtures. Proc. Joint Conf. on Thermodynamic and Transport Properties of Fluids, Inst. Mech. Eng. (London), 1958, pp. 133-141.

27. Mason, E. A., and Monchick, L.: Heat Conductivity of Polyatomic and Polar Gases. Jour. Chem. Phys., vol. 36, no. 6, Mar. 15, 1962, pp. 1622-1639.

TABLE I. - COMPARISON OF APPROXIMATE AND RIGOROUS VISCOSITIES OF

BINARY MIXTURES OF HELIUM, NEON, AND ARGON AT 20° C

Mole fraction		Viscosity, micropoise					
Helium	Neon	Rigorous	Approximate, eqs. (1) and (10)	Percent deviation	Approximate, eqs. (1) and (12)	Percent deviation	Experimental
0	1.0000	306.98	306.98	----	306.98	-----	[a]309.2
.2041	.7959	296.00	296.02	0.01	295.70	-0.10	[b]300.4
.2659	.7341	291.94	291.97	.01	291.55	-.13	[a]297.1
.5624	.4376	265.87	265.88	0	265.14	-.27	[a]270.2
.5781	.4219	264.11	264.12	0	263.37	-.28	[b]269.1
.7621	.2379	239.51	239.53	.01	238.82	-.29	[a]242.9
.7874	.2126	235.48	235.49	0	234.81	-.28	[b]240.3
1.0000	0	193.10	193.10	----	193.10	-----	[a]194.1
Helium	Argon						
0	1.0000	221.97	221.97	----	221.97	-----	[a]221.1
.3405	.6595	229.25	229.33	0.03	229.16	-0.04	[a]227.8
.3660	.6340	229.69	229.77	.03	229.58	-.05	[b]228.6
.3820	.6180	229.95	230.02	.03	229.83	-.05	[a]229.1
.4906	.5094	231.26	231.32	.03	231.05	-.09	[a]229.6
.5966	.4034	231.41	231.44	.01	231.09	-.14	[b]230.4
.7565	.2435	227.36	227.37	0	226.91	-.20	[b]227.0
1.0000	0	193.10	193.10	----	193.10	-----	[a]197.3
Neon	Argon						
0	1.0000	221.97	221.97	----	221.97	-----	[a]221.3
.2580	.7420	241.24	241.25	0	241.25	0	[a]240.1
.2962	.7038	244.28	244.29		244.29		[b]242.2
.3909	.6091	252.01	252.02		252.02		[a]250.4
.5382	.4618	264.59	264.60		264.60		[b]263.5
.7230	.2770	281.22	281.23		281.23		[b]281.1
.7320	.2680	282.05	282.06		282.06		[a]280.8
1.0000	0	306.98	306.98	----	306.98	-----	[a]309.2

[a]Ref. 21.

[b]Ref. 22.

TABLE II. - COMPARISON OF APPROXIMATE AND RIGOROUS VISCOSITIES OF TERNARY MIXTURES OF HELIUM, NEON, AND ARGON AT 20° C

Mole fraction			Viscosity, micropoise					
Helium	Neon	Argon	Rigorous	Approximate, eqs. (1) and (10)	Percent deviation	Approximate, eqs. (1) and (12)	Percent deviation	Experimental (ref. 22)
0.1754	0.5576	0.2670	273.43	273.46	0.01	273.27	-0.06	274.0
.1883	.3706	.4414	256.97	257.01	.02	256.86	-.04	255.7
.1983	.2166	.5851	243.96	244.01	.02	243.89	-.03	241.1
.2042	.4625	.3333	265.76	265.80	.02	265.60	-.06	265.5
.3594	.3193	.3213	257.17	257.21	.02	256.88	-.11	256.9
.4746	.3552	.1702	261.65	261.68	.01	261.17	-.18	262.9
.5175	.3210	.1615	258.62	258.65	.01	258.10	-.20	260.2
.5429	.2189	.2382	250.03	250.05	.01	249.56	-.19	250.4

TABLE III. - COMPARISON OF ERRORS OF VARIOUS APPROXIMATE

METHODS FOR VISCOSITY OF GAS MIXTURES

System	Type of error	Error, percent					
		Reference 3	Reference 4	Reference 5	Reference 13 (first approximation)	Present report, eqs. (1) and (10)	Present report, eqs. (1) and (12)
Helium-neon	Maximum	+3.6	+4.4	-0.3	+4.0	+0.01	-0.10
	Minimum	+1.1	+1.5	-.6	+1.3	0	-.29
	Root-mean-square average	2.9	3.5	.5	3.2	.01	.24
Helium-argon	Maximum	+3.4	+0.8	-0.2	+4.3	+0.03	-0.04
	Minimum	+1.1	-.2	-.6	+1.6	0	-.20
	Root-mean-square average	2.1	.4	.3	2.7	.02	.11
Neon-argon	Maximum	+0.6	-1.8	0	+0.2	0	0
	Minimum	+.2	-2.8	0	+.1	0	0
	Root-mean-square average	.5	2.4	0	.1	0	0
Helium-neon-argon	Maximum	+3.2	+0.9	0	+3.4	+0.02	-0.03
	Minimum	+1.0	-1.9	-.6	+1.0	+.01	-.20
	Root-mean-square average	2.2	1.2	.2	2.4	.02	.13
Overall	Maximum	+3.6	+4.4	0	+4.3	+0.03	0
	Minimum	+.2	-2.8	-.6	+.1	0	-.29
	Root-mean-square average	2.1	2.2	.3	2.4	.015	.14

TABLE IV. - COMPARISON OF APPROXIMATE AND RIGOROUS THERMAL CONDUCTIVITIES OF BINARY MIXTURES OF HELIUM, KRYPTON, AND XENON AT 29° C

Mole fraction		Thermal conductivity, microcal/(cm)(sec)(°K)					
Helium	Krypton	Rigorous	Approximate, eqs. (1) and (13)	Percent deviation	Approximate, eqs. (1) and (14)	Percent deviation	Experimental (ref. 24)
0	1.000	22.61	22.61	----	22.61	-----	23.2
.240	.760	54.70	54.71	0.02	54.72	0.04	54.1
.422	.578	88.73	88.74	.01	88.77	.05	88.8
.490	.510	104.65	104.67	.02	104.69	.04	103
.577	.423	128.51	128.53	.02	128.56	.04	128
.750	.250	192.80	192.82	.01	192.86	.03	193
.880	.120	265.41	265.43	.01	265.46	.02	261
1.000	0	367.13	367.13	----	367.13	-----	367
Helium	Xenon						
0	1.000	13.98	13.98	----	13.98	-----	14.27
.202	.798	38.81	38.83	0.05	38.84	0.08	35.7
.418	.582	76.38	76.43	.07	76.44	.08	71.7
.717	.283	166.16	166.23	.04	166.25	.05	153
.787	.213	199.31	199.38	.04	199.40	.05	188.2
1.000	0	367.13	367.13	----	367.13	-----	367
Krypton	Xenon						
0	1.000	13.98	13.98	----	13.98	-----	14.27
.110	.890	14.66	14.67	0.07	14.67	0.07	14.5
.158	.842	14.98	14.98	0	14.98	0	14.9
.276	.724	15.80	15.80	0	15.80	0	15.8
.510	.490	17.63	17.63	0	17.63	0	18.6
.785	.215	20.20	20.21	.05	20.21	.05	20.6
1.000	0	22.61	22.61	----	22.61	-----	23.2

TABLE V. - COMPARISON OF APPROXIMATE AND RIGOROUS THERMAL CONDUCTIVITIES OF TERNARY MIXTURES OF HELIUM, KRYPTON, AND XENON AT 29° C

Mole fraction			Thermal conductivity, microcal/(cm)(sec)(°K)					
Helium	Krypton	Xenon	Rigorous	Approximate, eqs. (1) and (13)	Percent deviation	Approximate, eqs. (1) and (14)	Percent deviation	Experimental (ref. 24)
0.219	0.086	0.695	42.16	42.18	0.05	42.19	0.07	40.5
.486	.057	.457	92.91	92.96	.05	92.97	.06	91.6
.709	.032	.259	164.01	164.07	.04	164.09	.05	158
.863	.016	.121	245.70	245.76	.02	245.78	.03	247
.248	.119	.633	46.97	47.00	.06	47.00	.06	45.5
.480	.082	.438	91.95	91.99	.04	92.01	.07	89.3
.742	.041	.217	178.98	179.05	.04	179.07	.05	171
.865	.021	.114	247.41	247.46	.02	247.48	.03	239
.215	.217	.568	42.98	43.00	.05	43.01	.07	41.2
.507	.136	.357	99.93	99.97	.04	99.99	.06	96.4
.706	.081	.213	164.66	164.71	.03	164.74	.05	152
.859	.039	.102	244.52	244.58	.02	244.60	.03	238
.227	.394	.379	47.00	47.01	.02	47.02	.04	45.0
.479	.266	.255	95.62	95.66	.04	95.68	.06	92.8
.729	.138	.133	177.22	177.27	.03	177.30	.05	174
.856	.073	.071	244.78	244.83	.02	244.85	.03	230
.245	.593	.162	52.82	52.83	.02	52.85	.06	53.3
.519	.378	.103	109.20	109.23	.03	109.26	.06	106
.743	.202	.055	186.87	186.91	.02	186.94	.04	184
.865	.106	.029	253.35	253.38	.01	253.41	.02	250

TABLE VI. - COMPARISON OF ERRORS OF VARIOUS APPROXIMATE METHODS FOR THERMAL CONDUCTIVITY OF MIXTURES OF MONATOMIC GASES

System	Type of error	Error, percent					
		Reference 6	Reference 10	Reference 7	Reference 13 (first approximation	Present report, eqs. (1) and (13)	Present report, eqs. (1) and (14)
Helium-krypton	Maximum	-2.5	+2.0	-3.2	+4.3	+0.02	+0.05
	Minimum	-5.4	+1.1	-4.6	+1.9	+.01	+.02
	Root-mean-square average	4.7	1.7	3.9	3.6	.02	.04
Helium-xenon	Maximum	-9.6	-6.5	-9.8	+3.4	+0.07	+0.08
	Minimum	-13.7	-12.3	-15.6	+2.2	+.04	+.05
	Root-mean-square average	11.7	10.1	13.0	2.8	.05	.07
Krypton-xenon	Maximum	+2.0	-0.5	+0.8	+2.0	+0.07	+0.07
	Minimum	+.8	-1.5	+.3	+.8	0	0
	Root-mean-square average	1.4	1.0	.6	1.4	.04	.04
Helium-krypton-xenon	Maximum	-3.8	-11.3	-3.2	+4.0	+0.06	+0.07
	Minimum	-12.7	-.2	-14.1	+1.7	+.01	+.02
	Root-mean-square average	8.9	6.5	9.7	3.2	.04	.05
Overall	Maximum	+2.0	+2.0	+0.8	+4.0	+0.07	+0.08
	Minimum	-13.7	-12.3	-15.6	+.8	0	0
	Root-mean-square average	8.1	6.0	8.7	3.0	.036	.050

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